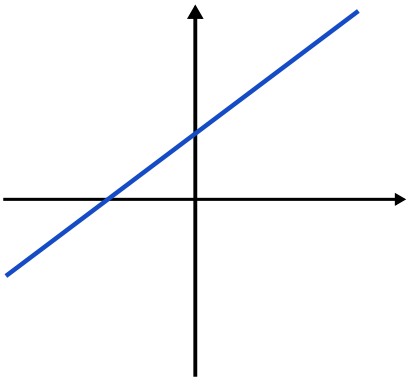
# Section 3.3

## Rules of Differentiation

Like in Section 2.6, we want a quick, efficient method for evaluating the limits of difference quotients. This way we do not need to compute a limit every time we wish to evaluate a derivative.

**Theorem 3.3.1** (Linear Functions)**.** If $f(x) = mx + b$ then $f'(c) = m$ for all real numbers $c$.



**Theorem 3.3.2** (Power Rule)**.** For any real number $n$,

$$\frac{d}{dx}x^n = nx^{n-1}.$$

For $n$ a positive integer:

**Theorem 3.3.3** (Constant Multiple Rule)**.** If $f$ is differentiable at $x$ and $c$ is some constant, then

$$\frac{d}{dx}\big(cf(x)\big) = cf'(x).$$

**Theorem 3.3.4** (Sum Rule)**.** If $f$ and $g$ are differentiable at $x$, then

$$\frac{d}{dx}\big(f(x) + g(x)\big) = f'(x) + g'(x).$$

Generalized Sum Rule:

**Example 3.3.5.** Compute the derivative for the following functions.

1. $y = x^5$

2. $f(x) = 8x^2$

3. $g(t) = 6\sqrt{t}$

4. $h(x) = 6x^5 - \frac{5}{2}x^2 + x + 5$

5. $f(t) = 6\sqrt{t} - 4\pi$

6. $g(w) = \begin{cases} w^2 + 1 & \text{if } w \leq 0 \\ 2w^2 + w + 1 & \text{if } w > 0 \end{cases}$

**What are Exponential Functions?** Let $f(x) = 2^x$. Compute

a) $f(2)$

b) $f(-2)$

c) $f(\frac{1}{3})$

d) $f(\pi)$

If $f(x) = b^x$ for $b > 0$, use the limit definition of the derivative to compute $f'(x)$.

**Definition 3.3.6** (Euler's Number or Napier's Constant)**.**
The number $e = 2.718281828459\ldots$ satisfies

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1.$$

**Theorem 3.3.7** (The Derivative of $e^x$)**.** The function $f(x) = e^x$ is differentiable for all real numbers and

$$f'(x) = e^x = f(x).$$

**Example 3.3.8.** Find the equation of the tangent line to $f(x) = \dfrac{e^x}{2}$ at $(0, \frac{1}{2})$.

**Definition 3.3.9** (Higher-Order Derivatives)**.** Assuming $f(x)$ can be differentiated as often as necessary, the *second derivative* of $f$ is

$$f''(x) = \frac{d^2}{dx^2} f(x) = \frac{d}{dx}(f'(x)).$$

For integers $n > 2$, the *nth derivative* of $f$ is

$$f^{(n)}(x) = \frac{d^n}{dx^n} f(x) = \frac{d}{dx}(f^{(n-1)}(x)).$$

We define $f^{(0)}(x) = f(x)$.

**Example 3.3.10.** Find the indicated higher-order derivatives.

1. $\dfrac{d^2}{dx^2}\left(x^4 - 2x^2\right)$

2. $\dfrac{d^5}{dx^5}\left(x^5\right)$

3. $g(t) = 3t^2 + 2e^t$; $g^{(100)}(t)$